Can the Presence of Autonomous Vehicles Worsen the Equilibrium State of Traffic Networks?

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***Abstract*— It is known that autonomous vehicles are capable of maintaining shorter headways and distances when they form platoons of vehicles. Thus, deployment of autonomous vehicles can result in roadway flow capacity increases in traffic networks. Consequently, it is envisioned that their deployment will boost the overall capacity of the network. In this paper, we consider a nonatomic routing game on a traffic network with inelastic (fixed) demands for the set of network O/D pairs, and study how replacing a fraction of regular (i.e. nonautonomous) vehicles by autonomous vehicles will affect the network total delay, under the assumption that the vehicles choose their routes selfishly. Using well known US bureau of public roads (BPR) traffic delay models, we show that the resulting Wardrop equilibrium is not necessarily unique even in its weak sense for networks with mixed autonomy. We derive the conditions under which the total network delay is guaranteed to not increase as a result of increasing the ratio of autonomous vehicles. However, we also show that when these conditions do not hold, counter intuitive behaviors might occur: the total delay can grow by increasing the fraction of autonomous vehicles in the network. In particular, we prove that for networks with a single O/D pair, if the road degree of capacity asymmetry (i.e. the ratio between the road capacity when all vehicles are regular and the road capacity when all vehicles are autonomous) is homogeneous, the total network delay is 1) unique, and 2) a nonincreasing continuous function of network autonomy fraction. We show that for heterogeneous degrees of capacity asymmetry, the total delay is not unique, and it can further grow when the fraction of autonomous vehicles increases. We demonstrate that similar behaviors may be observed in networks with multiple O/D pairs.**

I. Introduction

Autonomous vehicles technology has attracted significant attention as a result of its potentials for increasing safety, sustainability, and enhancing mobility. It has been shown in multiple works that autonomous vehicles can stabilize traffic flow and damp congestion shockwaves [1], [2], [3]. Moreover, there has been a recent focus on how to utilize vehicle autonomy and connectedness to remove signal lights from intersections and coordinate conflicting movements in order to maximize network throughput [4], [5], [6].

Additionally, autonomous vehicles can facilitate vehicle *platooning*. Vehicle platoons are groups of more than one vehicle, capable of maintaining shorter headways; as a result, platooning can lead to increases in the capacities of network links [7]. Such increases can be up to three–fold if all the vehicles are autonomous [7]. Before achieving full vehicle automation, transportation networks will go through a *transient* era, when both regular and autonomous vehicles coexist in the roadway. Therefore, it is crucial to study networks with mixed autonomy. In [8], the performance of networks with mixed autonomy was studied via simulations. In [1], deep reinforcement learning was utilized to predict the emergent behavior of traffic networks with mixed–autonomy. In [9], the capacities of the network links were modeled in a mixed–autonomy traffic network. Using affine delay functions, this model was used in [10] to compute the price of anarchy of traffic networks with mixed autonomy.

In this paper, we study how the introduction of autonomous vehicles in traffic networks will affect the equilibrium state of the network as compared to the case when all vehicles are regular (i.e. nonautonomous). In particular, given a fixed demand of vehicles, we study how replacing a fraction of vehicles with autonomous vehicles will affect the equilibrium assuming that all vehicles choose their routes in a selfish manner. This is of paramount importance since we need to predict how selfish behavior of the drivers in networks with mixed autonomy would change the behavior of the system. To this end, we model the selfish route choice behavior of vehicles as a nonatomic routing game [11] where vehicles choose their routes selfishly until a Wardrop equilibrium is achieved [12]. We model the network by a directed graph. We represent the delay along each network link, via commonly used BPR functions. We consider two classes of vehicles, regular and autonomous. For a given fixed demand profile, we study how increasing the autonomy fraction will affect the network at equilibrium.

We first show that the equilibrium may *not* be unique even in the weak sense of total link utilization. Then, we study networks with a single O/D pair and prove that if the degree of road capacity asymmetry is homogeneous throughout the network, the social or total delay of the network is a unique function of fraction of autonomous vehicles, which will be referred to as autonomy fraction throughout this paper. Further, the social delay is a continuous function and monotone nonincreasing in autonomy fraction. However, in networks with heterogeneous degrees of road asymmetry, we first show that the social delay is not unique. Then, we demonstrate that, surprisingly, increasing the autonomy fraction of the network may lead to an *increase* in the social delay. For networks with multiple O/D pairs, we show that similar complex behaviors may occur, namely increasing the autonomy fraction of a single O/D pair might worsen the total or social delay of the network. Our work in fact shows that traffic paradoxes similar to the well known Braess' Paradox [13] can occur

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in traffic networks with mixed vehicle autonomy due to the capacity increases provided by autonomous vehicles.

The organization of this paper is as follows. In Section II, we describe our notation and model. In Section III, we review some known results from routing games that we will further use. We discuss uniqueness of equilibrium in Section IV. Then, we analyze networks with a single O/D pair in Section V. In Section VI, we show the complexities caused by the presence of multiple O/D pairs. We conclude the paper and provide future directions in Section VII.

II. Nonatomic Selfish Routing

We model a traffic network by a directed graph $G = (N, L, W)$, where N and L are respectively the set of nodes and links in the network. Define $W = \{(o_1, d_1), (o_2, d_2), \cdots, (o_k, d_k)\}$ to be the set of origin–destination (O/D) pairs of the network. We assume that each link $l \in L$ joins two distinct nodes; thus, no self loops are allowed. A node $n \in N$ can appear in multiple O/D pairs. In a nonatomic selfish routing game with inelastic demands, each O/D pair has a fixed given nonzero demand and chooses some subset of paths from the set of possible paths from its origin to its destination to route its flow. Each O/D pair can decide on how much flow to send through each path such that the O/D pair minimizes its own delay. The delay of each path depends on how network links are shared among different O/D pairs. For each O/D pair $w = (o_i, d_i), 1 \leq i \leq K$, we let $\mathcal{P}_w$ denote the set of all possible network paths from o_i to d_i. We assume that the network topology is such that for each O/D pair $w \in W$, there exists at least one path from its origin to its destination, i.e. $\mathcal{P}_w \neq \emptyset$. We further let $\mathcal{P} = \cup_{w \in W} \mathcal{P}_w$ denote the set of all network paths.

For each O/D pair $w \in W$, and path $p \in \mathcal{P}_w$, let f_p be the flow of O/D pair w along path p. For each O/D pair w, let r_w be the given fixed demand of vehicles associated with w. Note that each path connects exactly one origin to one and only one destination; thereby, once a path is fixed, its origin and destination are uniquely determined. Consequently, there is no need to explicitly include path O/D pairs in the notation that is used for f_p. It is important to note that in our setting, each $w \in W$ has two classes of vehicles: regular and autonomous. Aligned with this, for each $w \in W$, define the autonomy fraction α_w to be the fraction of vehicles in r_w that are autonomous. Let r and α be the vectors of network demand and autonomy fractions respectively. Also for each path $p \in \mathcal{P}$, we use f_p^r and f_p^a to respectively denote the flow of regular and autonomous vehicles along path p. Note that for each $p \in \mathcal{P}$, $f_p = f_p^r + f_p^a$. The network flow vector f is a nonnegative vector of regular and autonomous flows along all network paths, i.e. $f = (f_p^r, f_p^a : p \in \mathcal{P})$. A nonnegative flow vector f is called feasible for a given network G if for each O/D pair $w \in W$,

$$\sum_{p \in \mathcal{P}_w} f_p^r = (1 - \alpha_w) r_w, \text{ and } \sum_{p \in \mathcal{P}_w} f_p^a = \alpha_w r_w, \tag{1a}$$

$$\forall p \in \mathcal{P} : f_p^r \geq 0, \text{ and } f_p^a \geq 0. \tag{1b}$$

For each link $l \in L$, f_l is the total flow of vehicles on l, i.e. $f_l = \sum_{p \in \mathcal{P}: l \in p} f_p$. Since we need to decompose the total link flow into regular and autonomous vehicles, we let f_l^r and f_l^a be the total flow of regular and autonomous vehicles along link l respectively, i.e. $f_l^r = \sum_{p \in \mathcal{P}: l \in p} f_p^r$ and $f_l^a = \sum_{p \in \mathcal{P}: l \in p} f_p^a$.

Note that if all vehicles are regular for all network O/D pairs $w \in W$, i.e. $\alpha_w = 0$, then, we only have a single class of regular vehicles, and for each path $p \in \mathcal{P}$, $f_p = f_p^r$. In this case, we can directly consider f_p rather than its decomposition into two classes of flows. The same argument holds for link flows, $f_l = f_l^r$ for all $l \in L$. In fact, if all vehicles are regular, our routing game reduces to a single class game.

$$\forall w \in W : \ \alpha_w = 0 \iff \forall p \in \mathcal{P} : \ f_p = f_p^r. \tag{2}$$

In order to be able to model the incurred delays when vehicles are routed throughout the network, it is assumed that each link $l \in L$ has a delay per unit of flow function $e_l(f_l^r, f_l^a) : \mathbb{R}^2 \to \mathbb{R}$. We assume that the delay per unit of flow for each path $p \in \mathcal{P}$ is obtained by the summation of the link delays over the links that form p

$$e_p(f) = \sum_{l \in L: l \in p} e_l(f_l^r, f_l^a). \tag{3}$$

Equation (3) implies that the delay of each path $p \in \mathcal{P}$, depends not only on the flows of regular and autonomous vehicles along path p, but also on the flows along other paths. The overall network delay or social delay is given by

$$J(f) = \sum_{p \in \mathcal{P}} f_p e_p(f). \tag{4}$$

A. Wardrop Equilibrium

It is well known in the transportation literature that if vehicles behave selfishly, a network is at an equilibrium if the well known Wardrop conditions hold [12]. The Wardrop conditions state that at equilibrium, no user has any incentive for unilaterally changing its path. This implies that for an equilibrium flow vector f, if there exists a path $p \in \mathcal{P}_w$ such that either $f_p^r \neq 0$ or $f_p^a \neq 0$, we must have that $e_p(f) \leq e_{p'}(f)$, for all $p' \in \mathcal{P}_w$.

Definition 1. A flow vector f is an equilibrium for a given network $G = (N, L, W)$ if and only if for every O/D pair $w \in W$ and every pair of paths $p, p' \in \mathcal{P}_w$,

$$f_p^r \left(e_p(f) - e_{p'}(f)\right) \leq 0, \tag{5a}$$

$$f_p^a \left(e_p(f) - e_{p'}(f)\right) \leq 0. \tag{5b}$$

Note that an implication of the above definition is that for each O/D pair $w \in W$, and any two paths $p, p' \in \mathcal{P}_w$ such that $f_p \neq 0$ and $f_{p'} \neq 0$, we must have that $e_p(f) = e_{p'}(f)$.

Definition 2. Given an equilibrium flow vector f for a network $G = (N, L, W)$, we define the delay of travel for each O/D pair $w \in W$ to be

$$e_w(f) := \min_{p \in \mathcal{P}_w} e_p(f). \tag{6}$$

Motivated by the above discussion, $e_w(f)$ is precisely the delay across all paths $p \in \mathcal{P}_w$ which have a nonzero flow. Moreover, the equilibrium condition implies that for a path $p \in \mathcal{P}_w$ with zero flow, we have $e_p(f) \geq e_w(f)$. It is worth mentioning that when there are no autonomous vehicles, since $f_p^r = f_p$ for all paths $p \in \mathcal{P}$, Conditions (5) reduce to:

$$\forall w \in W,\ \forall p, p' \in \mathcal{P}_w, \quad f_p\left(e_p(f) - e_{p'}(f)\right) \leq 0. \quad (7)$$

B. Delay Characterization

We first specify the structure of our delay functions. If there is only a single class of regular vehicles in the network, the US Bureau of Public Roads (BPR) [14] suggests the following form of delay functions.

Assumption 1. When network links are shared by only regular vehicles, the link delay functions $e_l(f_l)$ are of the following form

$$e_l(f_l) = a_l\left(1 + \gamma_l\left(\frac{f_l}{C_l}\right)^{\beta_l}\right), \quad (8)$$

where C_l is the capacity of link l, and a_l, γ_l, and β_l are nonnegative link parameters.

In practice, a_l is the free flow travel time along link l, and β_l is a positive integer ranging from 1 to 4. In order to characterize the link delay functions in networks with mixed autonomy, where we have two classes of vehicles, we first need to model the impact of autonomous vehicles on the link capacities. It was discussed in [9] that in networks with mixed autonomy, the link capacity C_l depends on the autonomy fraction of the link l defined as $\alpha_l := \frac{f_l^a}{f_l^a + f_l^r}$. We use $C_l(\alpha_l)$ to emphasize this dependence. Let m_l and M_l be the capacity of link l when all vehicles are regular and autonomous respectively. Since autonomous vehicles are capable of maintaining shorter headways, it is normally the case that $\mu_l = \frac{m_l}{M_l} \leq 1$ for each link $l \in L$. We will subsequently refer to μ_l as the link's degree of capacity asymmetry following [9]. When the two classes of regular and autonomous vehicles are present in the network, using the results in [9], we have

$$C_l(\alpha_l) = \frac{m_l M_l}{\alpha_l m_l + (1 - \alpha_l) M_l}. \quad (9)$$

We adopt this model throughout this paper. Since for each link $l \in L$, $\alpha_l = \frac{f_l^a}{f_l^a + f_l^r}$ and $f_l = f_l^a + f_l^r$, using (9), for networks with mixed autonomy, the delay function (8) can be modified as:

$$e_l(f_l^r, f_l^a) = a_l\left(1 + \gamma_l\left(\frac{f_l^r}{m_l} + \frac{f_l^a}{M_l}\right)^{\beta_l}\right). \quad (10)$$

III. Prior Work

A. Existence of Equilibrium

We state the following proposition from [15] which studies the conditions under which Wardrop Equilibrium exists for a multi class traffic network.

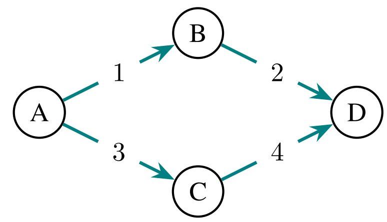

Fig. 1: A network with a single O/D pair and two paths.

Proposition 1. *Given a network $G = (N, L, W)$, if the link delay functions are continuous and monotone in the link flow of each class, then there exists at least one Wardrop equilibrium.*

Remark 1. Using (10), since our assumed delay functions are nonnegative, continuous, and monotone in the flow of each class, Proposition 1 implies that there always exists at least one Wardrop equilibrium for a routing game with mixed autonomy.

B. Equilibrium Uniqueness

In this part, we review known results regarding the uniqueness of Wardrop equilibrium. When multiple classes of vehicles are present in the network, the uniqueness of the equilibrium flow vector does not hold. However, uniqueness in a weak sense is known to hold [16].

Proposition 2. *For a general topology network G with multiple classes of vehicles on each O/D pair, if the delay functions are of the form (8), and the link capacities C_l are fixed and the same for all vehicle classes, for a given demand vector r, we have*

1) *The equilibrium is unique in a weak sense, i.e. for each link l the total flow f_l is unique.*
2) *For each O/D pair $w \in W$, the delay of travel $e_w(f)$ is unique for all Wardrop equilibrium flow vectors f.*

Note that if the conditions of Proposition 2 hold, the delay of travel for each O/D pair $e_w(f)$ is a well defined function of the network demand vector r and with a slight abuse of notation can be rewritten as $e_w(r)$.

Remark 2. A routing game that has only a single class of vehicles can be viewed as an instance of the games described in Proposition 2. Therefore, uniqueness in a the weak sense applies to games with a single class of vehicles too.

C. Social Delay Monotonicity

As we discussed above, in general, the equilibrium is not unique. However, if the conditions of Proposition 2 hold for a network, the social delay and the delay of travel for each O/D pair are unique. In particular, for a single class routing game on $G = (N, L, W)$, we recall the following from [17].

Proposition 3. *Consider a network $G = (N, L, W)$, where only one class of vehicles exist for each O/D pair $w \in W$. Assume that for each link $l \in L$, $e_l(.)$ is continuous, positive valued, and monotonically increasing. Then, for each O/D*

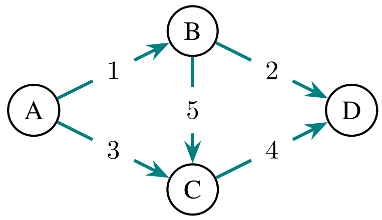

Fig. 2: A network with a single O/D pair and three paths from A to D.

pair $w \in W$, the delay of travel $e_w(r)$ is a continuous function of the demand vector r. Furthermore, $e_w(.)$ is nonincreasing in r_w when all other demands are fixed.

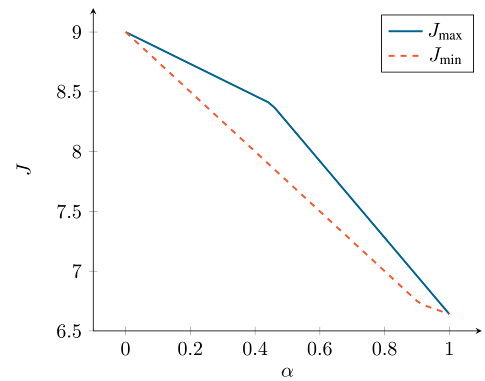

Fig. 3: Maximum and minimum delay for Example 2

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IV. Equilibrium Uniqueness

Now we study equilibrium uniqueness in the mixed autonomy setting. Using Remark 1, we know that there exists at least one equilibrium. However, since in our setting, for each link l, link capacity C_l depends on the autonomy ratio α_l, Proposition 2 does not apply. Indeed, we demonstrate through an example that the equilibrium is *not* unique even in the weak sense introduced in Proposition 2.

Example 1. Consider the network of Figure 1. Let p_1 and p_2 be the ABD and ACD paths respectively. For each link $l = 1, \cdots, 4$, let the link parameters be $\beta_l = 1, a_l = 1, m_l = 1$, and $M_l = 2$. Thus, for each link $l \in L$, the link delay function is $e_l(f_l^r, f_l^a) = 1 + f_l^r + \frac{f_l^a}{2}$. Assume that the demand from node A to D is $r = 2$, and the autonomy fraction $\alpha = 0.5$. Let f_1^r and f_1^a be the regular and autonomous vehicles flows along p_1, and f_2^r and f_2^a be the regular and autonomous flows along p_2. At equilibrium, since the network is symmetric, the case where one of the paths has a zero flow, and the other one has a nonzero flow cannot occur. Thus, at equilibrium, we must have

$$\begin{aligned} 2 + 2f_1^r + f_1^a &= 2 + 2f_2^r + f_2^a \\ f_1^r + f_2^r &= 1 \\ f_1^a + f_2^a &= 1 \\ f_1^r, f_1^a, f_2^r, f_2^a &\geq 0. \end{aligned}$$

Clearly, there is no unique solution to the above set of equations. Moreover, among the set of all possible equilibrium flow vectors, for each link, the maximum link flow at equilibrium is 1.25, whereas the minimum link flow is 0.75 at equilibrium, and the equilibrium is not unique even in the weak sense.

V. Networks with a single O/D pair

In this section, we study networks which have a single O/D pair. For such networks, since there is only one O/D pair, all paths originate from a common source o and end in a common destination d. Since W is singleton in this case, we omit the subscript w from r_w, e_w and α_w throughout this section. Note that when there is a single O/D pair, demand vector r and vector of autonomy fraction α are scalars.

Having observed that in the mixed–autonomy setting, the equilibrium is not unique, we study if the social delay is unique for all network equilibrium flow vectors. To this end, we use the notion of road degree of capacity asymmetry introduced in [10]. In the sequel, we consider the following two scenarios for investigating the properties of social delay.

1) homogeneous degrees of road capacity asymmetry, where μ_l is the same for all links, i.e. $\mu_l = \mu$, for all links $l \in L$, where μ is the common value of road capacity asymmetry.
2) heterogeneous degrees of capacity asymmetry, where μ_l varies on different links.

A. Homogeneous Degrees of Capacity Asymmetry

In this case, we can establish the uniqueness of social delay, and characterize the relationship between social delay and network autonomy fraction.

Theorem 1. *Given a network $G = (N, L, W)$ with a signle O/D pair and a homogeneous degree of capacity asymmetry μ on all its links, for any demand $r > 0$, we have*

1) *For a fixed autonomy fraction $0 \leq \alpha \leq 1$, the social delay $J(f)$ is unique for all Wardrop equilibrium flow vectors f.*
2) *If for each $0 \leq \alpha \leq 1$, we denote the common value of social delay in the above by $J(\alpha)$, then $J(.)$ is continuous and nonincreasing.*

Proof. Fix $r > 0$ and $0 \leq \alpha \leq 1$. Recalling Remark 1, we know that a Wardrop equilibrium exists. Let $f = (f_p^r, f_p^a : p \in \mathcal{P})$ be such an equilibrium where $f_p = f_p^a + f_p^r$ for each p in $\mathcal{P}$. Define $e_{\min}(f) := \min_{p \in \mathcal{P}} e_p(f)$. Since the network has only one O/D pair, and the delay associated with all paths with nonzero flows are all the same, the social delay can be obtained by $J(f) = re_{\min}(f)$. For each path $p \in \mathcal{P}$, define the fictitious single–class regular flow $\tilde{f}_p := f_p^r + \mu f_p^a$. We claim that the flow vector $\tilde{f} = (\tilde{f}_p : p \in \mathcal{P})$ is a Wardrop equilibrium for a routing game on G with a single class of regular vehicles and a total demand of $\tilde{r} = r(1 - \alpha) + r\alpha\mu$.

To see this, for each $p \in \mathcal{P}$, we let $\tilde{e}_p(\tilde{f})$ be the delay incurred by $\tilde{f}$ on path p and show that the relations (7) hold for $\tilde{f}$. Fix $p, p' \in \mathcal{P}$ and note that since f was a Wardrop equilibrium in the original setting, we must have $f_p^r(e_p(f) - e_{p'}(f)) \leq 0$, and $f_p^a(e_p(f) - e_{p'}(f)) \leq 0$. Multiplying the latter by the positive constant μ and adding the two inequalities, we have

$$\tilde{f}_p(e_p(f) - e_{p'}(f)) \leq 0, \qquad \forall p, p' \in \mathcal{P}. \tag{11}$$

Now, we claim that for all $p \in \mathcal{P}$, we have $e_p(f) = \tilde{e}_p(\tilde{f})$. Note that for each link $l \in L$, we have $\tilde{f}_l = f_l^r + \mu f_l^a$. Using the fact that $\mu = m_l/M_l$ for all links $l \in L$, we get

$$\begin{aligned} \tilde{e}_p(\tilde{f}) &= \sum_{l \in p} \left(a_l + \gamma_l \left(\frac{f_l^r + \frac{m_l}{M_l} f_l^a}{m_l} \right)^{\beta_l} \right) \\ &= \sum_{l \in p} \left(a_l + \gamma_l \left(\frac{f_l^r}{m_l} + \frac{f_l^a}{M_l} \right)^{\beta_l} \right) = e_p(f). \end{aligned} \tag{12}$$

Substituting into (11), we realize that

$$\tilde{f}_p(\tilde{e}_p(\tilde{f}) - \tilde{e}_{p'}(\tilde{f})) \leq 0, \qquad \forall p, p' \in \mathcal{P}, \tag{13}$$

which means that $\tilde{f}$ is an equilibrium flow vector. Clearly, the total demand of this new routing game is $\tilde{r} = \sum_{p \in \mathcal{P}} \tilde{f}_p = \sum_{p \in \mathcal{P}} f_p^r + \mu f_p^a = r(1-\alpha) + \mu \alpha r$. Moreover, define $\tilde{e}_{\text{min}}(\tilde{f})$ to be the minimum of $\tilde{e}_p(\tilde{f})$ among $p \in \mathcal{P}$. Since w is the single O/D pair of the network, $\tilde{e}_{\text{min}}(\tilde{f})$ is indeed equal to $\tilde{e}_w(\tilde{f})$, the travel delay of the single O/D pair of the network associated with $\tilde{f}$. Note that Proposition 2 implies that $\tilde{e}_{\text{min}}(\tilde{f})$ is a function of $\tilde{r}$ only. On the other hand, (12) implies that $\tilde{e}_{\text{min}}(\tilde{f}) = e_{\text{min}}(f)$. Putting these together, we realize that

$$J(f) = r e_{\text{min}}(f) = r \tilde{e}_{\text{min}}(\tilde{f}) = r \tilde{e}_w(\tilde{r}).$$

Note that the right hand side of the above identity does not depend on f, which establishes the proof of the first part. In fact, this shows that

$$J(\alpha) = r \tilde{e}_w(r(1-\alpha) + \alpha \mu r).$$

From Proposition 3, we know that $\tilde{e}_w(.)$ is continuous and nonincreasing. Also, since $\mu \leq 1$, the map $r \mapsto r(1-\alpha) + \alpha \mu r$ is continuous and nonincreasing. This completes the proof of the second part. $\square$

B. *Heterogeneous Degrees of Capacity Asymmetry*

Now, we allow μ_l to vary among the network links. We show that this makes the behavior of the system more complex. First, we show via the following example that the social delay is not necessarily unique in this case.

Example 2. Consider the network shown in Figure 2. Assume that $\gamma_l = 1, \beta_l = 1$, for $l = 1, 2, \cdots, 5$. Let the other link parameters be the following: $\{a_1 = 1, m_1 = 1, M_1 = 1\}$, $\{a_2 = 2, m_2 = 1, M_2 = 3\}$, $\{a_3 = 1, m_3 = 1, M_3 = 2\}$, $\{a_4 = 1, m_4 = 1, M_4 = 4\}$, and $\{a_5 = 3, m_5 = 1, M_5 = 3\}$. Moreover, let the total flow from origin A to destination D be 2. We computed the social delay for this network for any $\alpha > 0$ at different equilibria of the system, and we observed that the social delay is *not* unique. In particular,

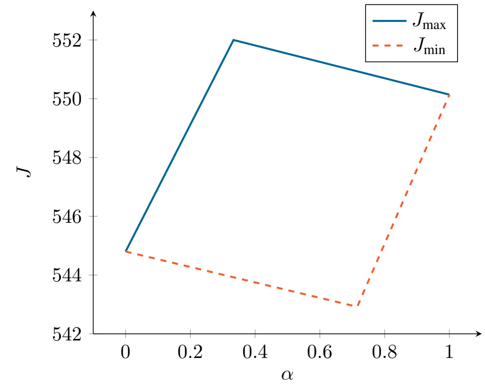

Fig. 4: Maximum and minimum social delays for Example 3.

Figure 3 shows the plots of the maximum and minimum social delay of system equilibria for every value of α. As Figure 3 shows, as soon as α starts to increase from 0, uniqueness of the social delay is lost. Once, $\alpha = 1$, the uniqueness of social delay is again preserved. This behavior implies that the change in the social delay due to increasing the autonomy ratio of the network is dependent on *which* equilibrium the system will be at.

Now, we study the effect of increasing network autonomy on the social delay. In the previous example, both the maximum and minimum social delays decreased as a function of α. But, is this necessarily the case? We use the following example to demonstrate that it might not be true in general, as increasing network autonomy may worsen the social delay in some networks.

Example 3. Consider the network in Figure 2 with the following parameters: $\{a_1 = 0, m_1 = 0.1, M_1 = \frac{1}{9}\}$, $\{a_2 = 50, m_2 = 1, M_2 = 1.1\}$, $\{a_3 = 50, m_3 = 1, M_3 = 1.1\}$, $\{a_4 = 0, m_4 = 0.1, M_4 = \frac{1}{9}\}$, $\{a_5 = 10, m_5 = 0.5, M_5 = 2\}$. Let the total flow r be 6. In this case, clearly, $\mu_l < 1$, for all links $l \in L$. Figure 4 shows the maximum and minimum social delay in this case for different values of α. Surprising, the maximum and minimum social delay have a nonmonotonic behavior. For certain values of α, even the minimum social delay is higher than the social delay at $\alpha = 0$. In particular, when all vehicles are autonomous, the social delay of the network is higher than that of the network when all vehicles are regular, i.e. $J(1) > J(0)$. This might be counter intuitive as we expect the network with full autonomy to have a lower social delay. However, this example shows that when capacity increases are heterogeneous across the network, the selfish behavior of vehicles might actually lead to worsening the social delay.

VI. Networks with Multiple O/D Pairs

So far, we have seen that even in a network with only one O/D pair, the introduction of autonomous vehicles can result in complex behaviors. Thus, it should be expected that a general network with multiple O/D pairs will exhibit similar counter intuitive behaviors. In the previous section,

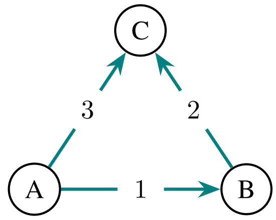

Fig. 5: A network with three O/D pairs.

we saw that the existence of a homogeneous degree of capacity asymmetry throughout the network is sufficient for guaranteeing improvements in the social delay by increasing the fraction of autonomous vehicles. We now show, via the following example, that this is not the case for networks with multiple O/D pairs.

Example 4. Consider the network shown in Figure 5. This network was first introduced in [18]. There are three O/D pairs, $W = \{$(A,B), (B,C), (A,C)$\}$. The total demand of the network O/D pairs are $r_{\text{AB}} = 1, r_{\text{AC}} = 20$, and $r_{\text{BC}} = 100$. Assume that $\gamma_l = 1$, $\beta_l = 1$, for all $l \in L$. Let the link parameters be $\{a_{\text{AB}} = 0, m_{\text{AB}} = 1, , M_{\text{AB}} = 2\}$, $\{a_{\text{BC}} = 0, m_{\text{BC}} = 1\}$, and $\{a_{\text{AC}} = 90, m_{\text{AC}} = 1\}$. Let the vehicles that wish to travel form A to C, and from B to C be all regular vehicles, i.e. $\alpha_{\text{AC}} = \alpha_{\text{BC}} = 0$. If the autonomy ratio of O/D pair AB is $\alpha_{AB} = 0$, the social delay is $J = 12384$. However, the social delay is equal to 12398 when $\alpha_{AB} = 0.1$. Therefore, the existence of autonomy for a certain O/D pair can result in worsening the social delay of the network.

It was shown in [19] that a decrease in demand of a particular O/D pair, might lead to an increase of delay along other O/D pairs and the social delay as result. In this example, we showed that similar behaviors can also emerge due to presence of selfishly routed autonomous vehicles. In fact, what we have shown so far is that the long known paradoxical traffic behavior resulting from constructing more roads or reducing demands can actually happen in networks with mixed autonomy due to the presence of autonomous vehicles.

VII. Conclusion and Future Work

In this paper, we studied how the coexistence of regular and autonomous vehicles in traffic networks will affect the network mobility when all vehicles select their routes selfishly. We compared the network social delay at a Wardrop equilibrium for networks with mixed vehicle autonomy with that of networks with only regular vehicles. Having shown that the equilibrium is not unique in some settings, we derived the conditions under which the social delay is unique, and it is further a nonincreasing and continuous function of the fraction of autonomous vehicles on the roadways. However, we showed that when these conditions do not hold, counter intuitive behaviors, such as the fact that increasing network autonomy fraction can worsen the network social delay, might occur. We believe that the insight provided by this work indicates that the mobility benefits of increasing autonomy in traffic networks are not immediate. For future steps, it is important to study the stability of the equilibria for networks with mixed autonomy. Once the stable system equilibria are characterized, control strategies must be developed for the system that are guaranteed to steer the system to the equilibria that have improved social delays. Therefore, revisiting routing and tolling strategies for such networks is imperative.

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